

Noise effects in a three-player prisoner's dilemma quantum game

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2008 J. Phys. A: Math. Theor. 41 435302

(<http://iopscience.iop.org/1751-8121/41/43/435302>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.152

The article was downloaded on 03/06/2010 at 07:17

Please note that [terms and conditions apply](#).

Noise effects in a three-player prisoner's dilemma quantum game

M Ramzan and M K Khan

Department of Physics, Quaid-i-Azam University, Islamabad 45320, Pakistan

E-mail: mramzan@phys.qau.edu.pk

Received 21 July 2008, in final form 2 September 2008

Published 30 September 2008

Online at stacks.iop.org/JPhysA/41/435302

Abstract

We study the three-player prisoner's dilemma game under the effect of decoherence and correlated noise. It is seen that the quantum player is always better off than the classical players. It is also seen that the game's Nash equilibrium does not change in the presence of correlated noise in contradiction to the effect of decoherence in the multiplayer case. Furthermore, it is shown that for maximum correlation the game does not behave as a noiseless game and the quantum player is still better off for all values of the decoherence parameter p which is not possible in the two-player case. In addition, the payoffs reduction due to decoherence is controlled by the correlated noise throughout the course of the game.

PACS numbers: 03.67.-a, 02.20.-a, 42.50.Lc

1. Introduction

Quantum entanglement provides a fundamental potential resource for communication and information processing and is one of the key quantitative notions of the intriguing field of quantum information theory and quantum computation. A quantum superposition state decays into a classical, statistical mixture of states through a decoherence process which is caused by entangling interactions between the system and its environment [1]. Superposition of quantum states, however, are very fragile and easily destroyed by the decoherence processes. Such uncontrollable influences cause noise in the communication or errors in the outcome of a computation, and thus reduce the advantages of quantum information methods. However, in a more realistic and practical situation, decoherence caused by an external environment is inevitable. Therefore, the influence of an external environmental system on the entanglement cannot be ignored. Novel research has been carried out to study the quantum communication channels. Macchiavello and Palma [2] have developed the theory of quantum channels to encompass memory effects. In real-world applications the assumption of having uncorrelated

noise channels cannot be fully justified. However, quantum computing in the presence of noise is possible with the use of decoherence free subspaces [3] and the quantum error correction [4].

The application of mathematical physics to economics has seen a recent development in the form of quantum game theory. Two-player quantum games have attracted a lot of interest in recent years [5–7]. A number of authors have investigated the quantum prisoner’s dilemma game [8–10]. A detailed description of quantum game theory can be found in [11–16]. There have been remarkable advances in the experimental realization of quantum games such as prisoner’s dilemma [17, 18]. The prisoner’s dilemma game is a widely known example in classical game theory. The quantum version of the prisoner’s dilemma has been experimentally demonstrated using a nuclear magnetic resonance (NMR) quantum computer [18]. Recently, Prevedel *et al* have experimentally demonstrated the application of a measurement-based protocol [19]. They realized a quantum version of the prisoner’s dilemma game based on the entangled photonic cluster states. It was the first realization of a quantum game in the context of one-way quantum computing. Studies concerning the quantum games in the presence of decoherence and correlated noise have produced interesting results. Chen *et al* [20] have shown that in the case of the two-player prisoner’s dilemma game, the Nash equilibria are not changed by the effect of decoherence in a maximally entangled case. Nawaz and Toor [21] have studied quantum games under the effect of correlated noise by taking a particular example of the phase-damping channel. They have shown that the quantum player outperforms the classical players for all values of the decoherence parameter p . They have also shown that for maximum correlation the effects of decoherence diminish and it behaves as a noiseless game. Recently, we have investigated different quantum games under different noise models and found interesting results [22]. More recently, Gawron *et al* [23] have studied the noise effects in the quantum magic squares game. They have shown that the probability of success can be used to determine characteristics of quantum channels. The investigation of multiplayer quantum games in a multi-qubit system could be of much interest and significance. In recent years, quantum games with more than two players were investigated [24–27]. Such games can exhibit certain forms of pure quantum equilibrium that have no analog in classical games, or even in two-player quantum games. Recently, Cao *et al* [28] have investigated the effect of quantum noise on a multiplayer prisoner’s dilemma quantum game. They have shown that in a maximally entangled case a special Nash equilibrium appears for a specific range of the quantum noise parameter (the decoherence parameter). However, no attention has yet been given to the multiplayer quantum games under the effect of correlated noise, which is the main focus of this paper.

In this paper, we investigate the three-player prisoner’s dilemma quantum game under the effect of decoherence and correlated noise in a three-qubit system. We have considered a dephasing channel parameterized by the memory factor μ which measures the degree of correlations. By exploiting the initial state and measurement basis entanglement parameters, $\gamma \in [0, \pi/2]$ and $\delta \in [0, \pi/2]$, we study the role of the decoherence parameter $p \in [0, 1]$ and memory parameter $\mu \in [0, 1]$ on the three-player prisoner’s dilemma quantum game. Here, $\delta = 0$ means that the measurement basis are unentangled and $\delta = \pi/2$ means that it is maximally entangled. Similarly, $\gamma = 0$ means that the game is initially unentangled and $\gamma = \pi/2$ means that it is maximally entangled. Whereas the lower and upper limits of p correspond to a fully coherent and fully decohered system, respectively. Similarly, the lower and upper limits of μ correspond to a memoryless and maximum memory (degree of correlation) cases, respectively. It is seen that in contradiction to the two-player prisoner’s dilemma quantum game, in the three-player game, the quantum player can outperform the classical players for all values of the decoherence parameter p for the maximum degree of

Table 1. The payoff matrix for the three-player prisoner’s dilemma game where the first number in the parentheses denotes the payoff of Alice, the second number denotes the payoff of Bob and the third number denotes the payoff of Charlie.

		<u>Charlie C</u>		<u>Charlie D</u>			
		<u>Bob</u>		<u>Bob</u>			
<u>Alice</u>	<u>C</u>	(3,3,3)	(2,5,2)	<u>Alice</u>	<u>C</u>	(2,2,5)	(0,4,4)
	<u>D</u>	(5,2,2)	(4,4,0)		<u>D</u>	(4,0,4)	(1,1,1)

correlations (i.e. memory parameter $\mu = 1$). In comparison to the two-player situation, the three-player game does not become noiseless and the quantum player still remains superior to the classical ones for an entire range of the decoherence parameter, p , in the memoryless case, i.e. $\mu = 0$. It is shown that the payoffs reduction due to decoherence is controlled by the memory parameter μ throughout the course of the game. It is also shown that the Nash equilibrium of the game does not change under the correlated noise in contradiction to the case of decoherence effects as investigated by Cao *et al* [28].

2. Three-player prisoner’s dilemma game

Properties of the two-player quantum games have been discussed extensively [11–13, 29], however, not much attention has been given to the multiplayer quantum games. Study of the multiplayer games may exhibit interesting results in comparison to the two-player games. The three-player prisoners’ dilemma is similar to the two-player situation except that Alice, Bob and a third player, Charlie, join the game. The three players are arrested under the suspicion of robbing a bank. Similar to the two-player case, they are interrogated in separate cells without communicating with each other. The two possible moves for each prisoner are to cooperate (C) or to defect (D). The payoff table for the three-player prisoner’s dilemma is shown in table 1. The game is symmetric for the three players, and the strategy D dominates the strategy C for all of them. Since the selfish players prefer to choose D as the optimal strategy, the unique Nash equilibrium is (D, D, D) with payoffs $(1, 1, 1)$. This is a Pareto inferior outcome, since (C, C, C) with payoffs $(3, 3, 3)$ would be better for all the three players. This situation is the very catch of the dilemma and is similar to the two-player version of this game. The dilemma of this game can be resolved in its quantum version. Du *et al* [25] investigated the three-player quantum prisoner’s dilemma game with a certain strategic space. They found a Nash equilibrium that can remove the dilemma in the classical game when the game’s state is maximally entangled. This particular Nash equilibrium remains to be a Nash equilibrium even for the non-maximally entangled cases. However, their calculations for the expected payoffs of the players comprise product measurement basis for the arbiter of the game. Here in our model we use the entangled measurement basis for the arbiter of the game to perform measurement. In addition, we include the effect of decoherence and correlated noise in the three-players settings.

3. Time-correlated dephasing channel

Quantum information is encoded in qubits during its transmission from one party to another and requires a communication channel. In a realistic situation, the qubits have a nontrivial

dynamics during transmission because of their interaction with the environment. Therefore, Bob may receive a set of distorted qubits because of the disturbing action of the channel. Studies on quantum channels have attracted a lot of attention in recent years [2, 30]. Early work in this direction was devoted mainly to memoryless channels for which consecutive signal transmissions through the channel are not correlated. In the correlated channels (channels with the memory), the noise acts on consecutive uses of channels. We consider here the noise model based on the time-correlated dephasing channel. In the operator sum representation, the dephasing process can be expressed as [31]

$$\rho_f = \sum_{i=0}^1 A_i \rho_{in} A_i^\dagger, \tag{1}$$

where

$$\begin{aligned} A_0 &= \sqrt{1 - \frac{p}{2}} I \\ A_1 &= \sqrt{\frac{p}{2}} \sigma_z \end{aligned} \tag{2}$$

are the Kraus operators, I is the identity operator, σ_z is the Pauli matrix and p is the decoherence parameter. Let N qubits are allowed to pass through such a channel then equation (1) becomes [32]

$$\rho_f = \sum_{k_1, \dots, k_n=0}^1 (A_{k_n} \otimes \dots \otimes A_{k_1}) \rho_{in} (A_{k_1}^\dagger \otimes \dots \otimes A_{k_n}^\dagger). \tag{3}$$

Now if the noise is correlated with the memory of degree μ , then the action of the channel on the two consecutive qubits is given by the Kraus operators [2]

$$A_{ij} = \sqrt{p_i[(1 - \mu)p_j + \mu\delta_{ij}]} \sigma_i \otimes \sigma_j, \tag{4}$$

where σ_i and σ_j are usual Pauli matrices with indices i and j run from 0 to 3 and μ is the memory parameter. The above expression means that with the probability $(1 - \mu)$ the noise is uncorrelated whereas with the probability μ the noise is correlated. Physically the parameter μ is determined by the relaxation time of the channel when a qubit passes through it. In order to remove correlations, one can wait until the channel has relaxed to its original state before sending the next qubit. However, this may lower the rate of information transfer. The Kraus operators for the three-qubit system can be written as [33]

$$A_{ijk} = \sqrt{[(1 - \mu)p_i + \mu\delta_{ij}][(1 - \mu)p_j + \mu\delta_{jk}]p_k} \sigma^i \otimes \sigma^j \otimes \sigma^k, \tag{5}$$

where i, j, k are 0 or 3. The memory parameter μ is contained in the probabilities A_{ijk} , which determines the probability of the errors $\sigma^i \otimes \sigma^j \otimes \sigma^k$. Recalling that $(1 - \mu)$ is the probability of independent errors on two consecutive qubits and μ is the probability of identical errors. The sum of probabilities of all types of errors on the three qubits add to unity as we expect,

$$\sum_{i,j,k} [(1 - \mu)^2 A_i A_j A_k + 2\mu(1 - \mu) A_i A_j + \mu^2 A_i] = 1. \tag{6}$$

It is necessary to consider the performance of the channel for arbitrary values of μ to reach a compromise between various factors which determine the final rate of information transfer. Thus in passing through the channel any two consecutive qubits undergo random independent (uncorrelated) errors with the probability $(1 - \mu)$ and identical (correlated) errors with the probability μ . This should be the case if the channel has a memory depending on its relaxation time and if we stream the qubits through it.

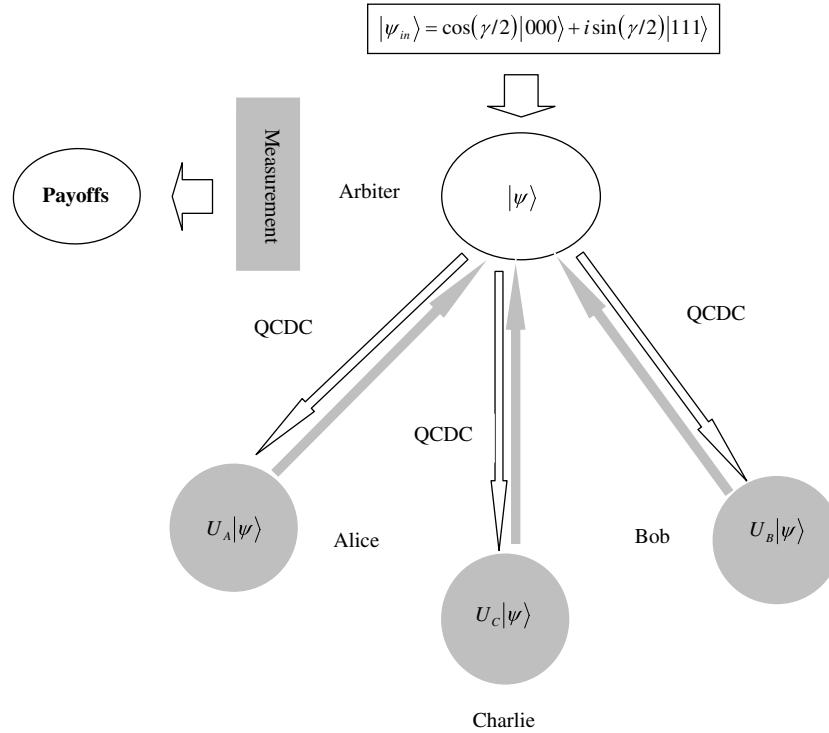


Figure 1. Schematic diagram of the model.

4. The model

In our model, Alice, Bob and Charlie, each uses individual channels to communicate with the arbiter of the game. The two uses of the channel, i.e. the first passage (from the arbiter) and the second passage (back to the arbiter) are correlated as depicted in figure 1. We consider that the initial entangled state is prepared by the arbiter and passed on to the players through a quantum correlated dephasing channel (QCDC). On receiving the quantum state, the players apply their local operators (strategies) and return it back to the arbiter via QCDC. Then, the arbiter performs the measurement and announces their payoffs. Let us consider that the three players Alice, Bob and Charlie be given the following initial quantum state:

$$|\psi_{in}\rangle = \cos \frac{\gamma}{2} |000\rangle + i \sin \frac{\gamma}{2} |111\rangle, \tag{7}$$

where $0 \leq \gamma \leq \pi/2$ corresponds to the entanglement of the initial state. The players can locally manipulate their individual qubits. The strategies of the players can be represented by the unitary operator U_i of the form [22].

$$U_i = \cos \frac{\theta_i}{2} R_i + \sin \frac{\theta_i}{2} P_i, \tag{8}$$

where $i = 1, 2$ or 3 and R_i, P_i are the unitary operators defined as

$$\begin{aligned} R_i |0\rangle &= e^{i\alpha_i} |0\rangle, & R_i |1\rangle &= e^{-i\alpha_i} |1\rangle \\ P_i |0\rangle &= e^{i(\frac{\pi}{2}-\beta_i)} |1\rangle, & P_i |1\rangle &= e^{i(\frac{\pi}{2}+\beta_i)} |0\rangle, \end{aligned} \tag{9}$$

where $0 \leq \theta_i \leq \pi$, and $-\pi \leq \{\alpha_i, \beta_i\} \leq \pi$. The application of the local operators of the players transforms the initial state given in equation (7) to

$$\rho_f = (U_1 \otimes U_2 \otimes U_3) \rho_{\text{in}} (U_1 \otimes U_2 \otimes U_3)^\dagger, \quad (10)$$

where $\rho_{\text{in}} = |\psi_{\text{in}}\rangle \langle \psi_{\text{in}}|$ is the density matrix for the quantum state. The operators used by the arbiter to determine the payoffs for Alice, Bob and Charlie are

$$P^k = \$_{000}^k P_{000} + \$_{001}^k P_{001} + \$_{110}^k P_{110} + \$_{010}^k P_{010} \\ + \$_{101}^k P_{101} + \$_{011}^k P_{011} + \$_{100}^k P_{100} + \$_{111}^k P_{111}, \quad (11)$$

where $k = A, B$ or C and

$$\begin{aligned} P_{000} &= |\psi_{000}\rangle \langle \psi_{000}|, & |\psi_{000}\rangle &= \cos \frac{\delta}{2} |000\rangle + i \sin \frac{\delta}{2} |111\rangle \\ P_{111} &= |\psi_{111}\rangle \langle \psi_{111}|, & |\psi_{111}\rangle &= \cos \frac{\delta}{2} |111\rangle + i \sin \frac{\delta}{2} |000\rangle \\ P_{001} &= |\psi_{001}\rangle \langle \psi_{001}|, & |\psi_{001}\rangle &= \cos \frac{\delta}{2} |001\rangle + i \sin \frac{\delta}{2} |110\rangle \\ P_{110} &= |\psi_{110}\rangle \langle \psi_{110}|, & |\psi_{110}\rangle &= \cos \frac{\delta}{2} |110\rangle + i \sin \frac{\delta}{2} |001\rangle \\ P_{010} &= |\psi_{010}\rangle \langle \psi_{010}|, & |\psi_{010}\rangle &= \cos \frac{\delta}{2} |010\rangle - i \sin \frac{\delta}{2} |101\rangle \\ P_{101} &= |\psi_{101}\rangle \langle \psi_{101}|, & |\psi_{101}\rangle &= \cos \frac{\delta}{2} |101\rangle - i \sin \frac{\delta}{2} |010\rangle \\ P_{011} &= |\psi_{011}\rangle \langle \psi_{011}|, & |\psi_{011}\rangle &= \cos \frac{\delta}{2} |011\rangle - i \sin \frac{\delta}{2} |100\rangle \\ P_{100} &= |\psi_{100}\rangle \langle \psi_{100}|, & |\psi_{100}\rangle &= \cos \frac{\delta}{2} |100\rangle - i \sin \frac{\delta}{2} |011\rangle, \end{aligned} \quad (12)$$

where $0 \leq \delta \leq \pi/2$ and $\$_{lmn}^k$ are elements of the payoff matrix as given in table 1. Since quantum mechanics is a fundamentally probabilistic theory, the strategic notion of the payoff is the expected payoff. The players after their actions forward their qubits to the arbiter of the game for the final projective measurement in the computational basis (see equation (12)). The arbiter of the game finally determines their payoffs (see figure 1). The payoffs for the players can be obtained as the mean values of the payoff operators as

$$\$_k(\theta_i, \alpha_i, \beta_i) = \text{Tr}(P^k \rho_f), \quad (13)$$

where Tr represents the trace of the matrix. Using equations (5)–(13), the payoffs for the three players can be obtained as

$$\begin{aligned} \$_k(\theta_i, \alpha_i, \beta_i) &= c_1 c_2 c_3 [\eta_1 \$_{000}^k + \eta_2 \$_{111}^k + (\$_{000}^k - \$_{111}^k) \mu_p^{(1)} \mu_p^{(2)} \xi \cos 2(\alpha_1 + \alpha_2 + \alpha_3)] \\ &+ s_1 s_2 s_3 [\eta_2 \$_{000}^k + \eta_1 \$_{111}^k - (\$_{000}^k - \$_{111}^k) \mu_p^{(1)} \mu_p^{(2)} \xi \cos 2(\beta_1 + \beta_2 + \beta_3)] \\ &+ c_1 c_2 s_3 [\eta_1 \$_{001}^k + \eta_2 \$_{110}^k + (\$_{001}^k - \$_{110}^k) \mu_p^{(1)} \mu_p^{(2)} \xi \cos 2(\alpha_1 + \alpha_2 - \beta_3)] \\ &+ s_1 s_2 c_3 [\eta_2 \$_{001}^k + \eta_1 \$_{110}^k - (\$_{001}^k - \$_{110}^k) \mu_p^{(1)} \mu_p^{(2)} \xi \cos 2(\beta_1 + \beta_2 - \alpha_3)] \\ &+ s_1 c_2 c_3 [\eta_1 \$_{100}^k + \eta_2 \$_{011}^k + (\$_{100}^k - \$_{011}^k) \mu_p^{(1)} \mu_p^{(2)} \xi \cos 2(\alpha_2 + \alpha_3 - \beta_1)] \\ &+ c_1 s_2 s_3 [\eta_2 \$_{100}^k + \eta_1 \$_{011}^k - (\$_{100}^k - \$_{011}^k) \mu_p^{(1)} \mu_p^{(2)} \xi \cos 2(\beta_2 + \beta_3 - \alpha_1)] \\ &+ s_1 c_2 s_3 [\eta_1 \$_{101}^k + \eta_2 \$_{010}^k + (\$_{101}^k - \$_{010}^k) \mu_p^{(1)} \mu_p^{(2)} \xi \cos 2(\beta_1 + \beta_3 - \alpha_2)] \\ &+ c_1 s_2 c_3 [\eta_2 \$_{101}^k + \eta_1 \$_{010}^k - (\$_{101}^k - \$_{010}^k) \mu_p^{(1)} \mu_p^{(2)} \xi \cos 2(\alpha_1 + \alpha_3 - \beta_2)] \end{aligned}$$

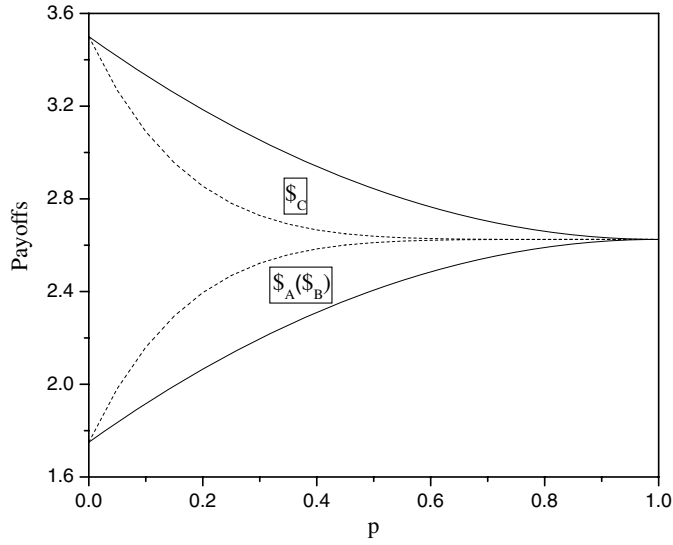


Figure 2. Players payoffs as a function of the decoherence parameter p for the dephasing channel are plotted for the quantum prisoner’s dilemma game, with memory parameter $\mu = 1$ (solid lines), $\mu = 0$ (dotted lines). $\$_{A(B)}$ are payoffs of the classical players (Alice/Bob) while $\$_C$ represents the payoff of the quantum player (Charlie). The other parameters are $\theta_1 = \theta_2 = \theta_3 = \pi/2, \beta_1 = \beta_2 = \alpha_1 = \alpha_2 = 0, \delta = \gamma = \pi/2$, and $\alpha_3 = \pi/2, \beta_3 = \pi/2$ are the optimal strategies of Charlie.

$$\begin{aligned}
 & + \frac{\mu_p^{(1)}}{8} (\cos^2(\delta/2) - \sin^2(\delta/2)) [\$_{000}^k - \$_{111}^k \\
 & - \$_{001}^k + \$_{110}^k - \$_{010}^k + \$_{101}^k + \$_{011}^k - \$_{100}^k] \\
 & \times \sin(\gamma) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) \cos(\alpha_1 + \alpha_2 + \alpha_3 - \beta_1 - \beta_2 - \beta_3) \\
 & + [[\$_{000}^k - \$_{111}^k] \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) \cos(\alpha_1 + \alpha_2 + \alpha_3 - \beta_1 - \beta_2 - \beta_3) \\
 & + [\$_{110}^k - \$_{001}^k] \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) \cos(\alpha_1 + \alpha_2 - \alpha_3 + \beta_1 + \beta_2 - \beta_3) \\
 & + [\$_{010}^k - \$_{101}^k] \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) \cos(\alpha_1 - \alpha_2 + \alpha_3 + \beta_1 - \beta_2 + \beta_3) \\
 & + [\$_{100}^k - \$_{011}^k] \sin(\delta) \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) \cos(\alpha_1 - \alpha_2 - \alpha_3 + \beta_1 \\
 & - \beta_2 - \beta_3)] \left[\frac{\mu_p^{(2)}}{8} (\cos^2(\gamma/2) - \sin^2(\gamma/2)) \right], \tag{14}
 \end{aligned}$$

where

$$\begin{aligned}
 \mu_p^{(j)} &= (1 - p_j)(1 - 2p_j + 4\mu_j p_j - 2\mu_j^2 p_j + p_j^2 - 2\mu_j p_j^2 + \mu_j^2 p_j^2) \\
 \eta_1 &= \cos^2(\gamma/2) \cos^2(\delta/2) + \sin^2(\gamma/2) \sin^2(\delta/2) \\
 \eta_2 &= \sin^2(\gamma/2) \cos^2(\delta/2) + \sin^2(\delta/2) \cos^2(\gamma/2) \\
 \xi &= \frac{1}{2} \sin(\delta) \sin(\gamma), \quad c_i = \cos^2 \frac{\theta_i}{2}, \quad s_i = \sin^2 \frac{\theta_i}{2}, \tag{15}
 \end{aligned}$$

where $j = 1$ or 2 . The payoffs for the three players can be found by substituting the appropriate values for $\$_{lmn}^k$ into equation (14). The elements of the classical payoff matrix for the prisoner’s dilemma game are given in table 1. The payoff matrix under decoherence

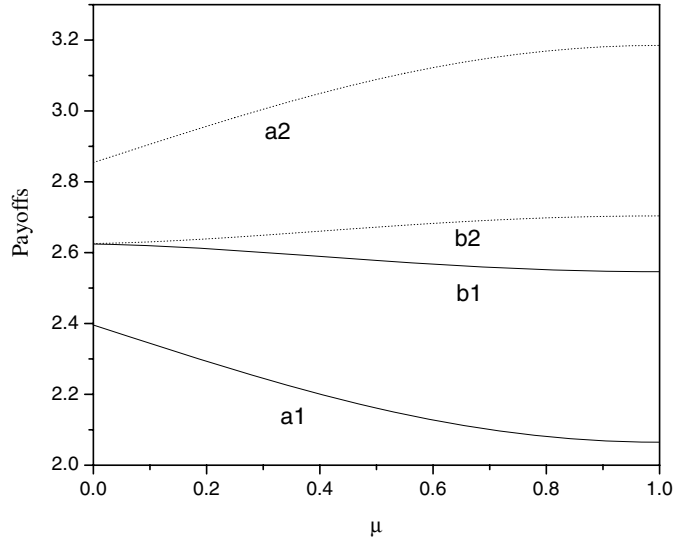


Figure 3. Payoffs of the classical players (Alice/Bob) and the quantum player (Charlie) are plotted as a function of memory parameter μ . a_1 and a_2 are payoffs of the classical players for values of the decoherence parameter $p = 0.7$ and $p = 0.3$, respectively. c_1 and c_2 are payoffs of quantum player for $p = 0.7$ and $p = 0.3$, respectively. The other parameters are $\theta_1 = \theta_2 = \theta_3 = \pi/2$, $\beta_1 = \beta_2 = \alpha_1 = \alpha_2 = 0$, $\delta = \gamma = \pi/2$, and $\alpha_3 = \pi/2$, $\beta_3 = \pi/2$ are the optimal strategies of Charlie.

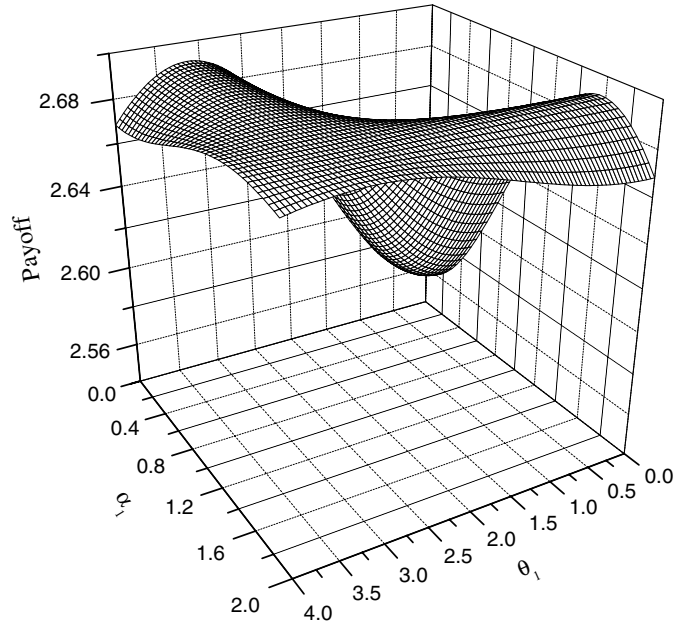


Figure 4. Alice's payoff is plotted as a function of her strategies α_1 and θ_1 with $\theta_2 = \theta_3 = \pi/2$, $\alpha_2 = \alpha_3 = \beta_1 = \beta_2 = \beta_3 = 0$, $\delta = \gamma = \pi/2$ and $p = \mu = 0.3$.

can be obtained by setting $\mu = 0$, i.e. by setting $\mu_p^{(j)} = (1 - p_j)^3$ in equation (15). It is important to mention that for p and μ we mean $p_1 = p_2 = p$ and $\mu_1 = \mu_2 = \mu$

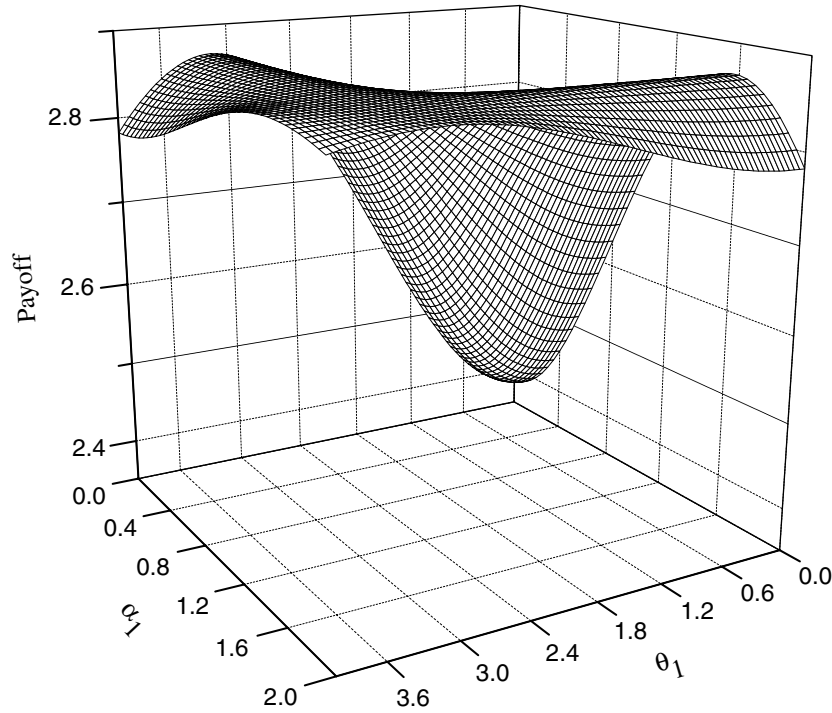


Figure 5. Alice's payoff is plotted as a function of her strategies α_1 and θ_1 with $\theta_2 = \theta_3 = \pi/2$, $\alpha_2 = \alpha_3 = \beta_1 = \beta_2 = \beta_3 = 0$, $\delta = \gamma = \pi/2$ and $p = \mu = 0.7$.

unless otherwise specified. Our results are consistent with [25, 27] and can be verified from equation (14) when all the three players resort to their Nash equilibrium strategies. It can be seen that the decoherence causes a reduction in the payoffs of the players in the memoryless case (see equation (14)). We consider here that Alice and Bob are restricted to play classical strategies, i.e., $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$, whereas Charlie is allowed to play the quantum strategies as well. It is shown that the quantum player outperforms the classical players for all values of the decoherence parameter p for an entire range of the memory parameter μ . Under these circumstances, it is seen that in contradiction to the two-player prisoner's dilemma quantum game, for maximum degree of correlations the effect of decoherence survives and it does not behave as a noiseless game. It can be seen that the memory compensates the payoffs reduction due to decoherence. Furthermore, it is shown that the memory has no effect on the Nash equilibrium of the game. Alice's best strategy ($\alpha_1 = \theta_1 = \pi/2$, and $\beta_1 = 0$) remains her best strategy throughout the course of the game. This implies that the correlated noise has no effect on the Nash equilibrium of the game.

5. Results and discussions

To analyze the effects of correlated noise (memory) and decoherence on the dynamics of the three-player prisoner's dilemma quantum game. We consider the restricted game scenario where Alice and Bob are allowed to play the classical strategies, i.e., $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$, whereas Charlie is allowed to play the quantum strategies. In figure 2, we have plotted the player's payoffs as a function of the decoherence parameter p for the dephasing channel. It is

seen that the quantum player outcores the classical players for all values of the decoherence parameter p for the memoryless ($\mu = 0$) case. It is shown that even for a maximum degree of memory, i.e. $\mu = 1$, the quantum player can outperform the classical players, which is in contradiction to the two-player prisoner's dilemma quantum game. In addition, the decoherence effects persist for maximum correlation and it does not behave as a noiseless game, contrary to the two-player case. In figure 3, we have plotted payoffs of the classical and the quantum players as a function of the memory parameter μ for $p = 0.3$ and 0.7 , respectively. It is seen that memory compensates the payoffs reduction due to decoherence. In figures 4 and 5, we have plotted Alice's payoff as a function of her strategies α_1 and θ_1 for $p = \mu = 0.3$ and $p = \mu = 0.7$, respectively. It can be seen that the memory has no effect on the Nash equilibrium of the game. It is evident from figures 4 and 5 that the best strategy for Alice is $\alpha_1 = \theta_1 = \pi/2$, and $\beta_1 = 0$. It remains her best strategy for the full range of the decoherence parameter p and the memory parameter μ , throughout the course of the game. Therefore, it can be inferred that correlated noise has no effect on the Nash equilibrium of the game. In comparison to the investigations of Cao *et al* [28], it is shown that the new Nash equilibrium, appearing for a specific range of the decoherence parameter p , disappears under the effect of correlated noise. As it can be seen that for the entire range of the decoherence parameter p and the memory parameter μ , the Nash equilibrium of the game does not change (see figures 4 and 5). Furthermore, it can also be seen that the payoffs of the players are increased with the addition of the correlated noise as can be seen from figures 4 and 5, respectively, for the entire ranges of the decoherence and the memory parameters.

6. Conclusions

We present a quantization scheme for the three-player prisoner's dilemma game under the effect of decoherence and correlated noise. We study the effects of decoherence and correlated noise on the game dynamics. We consider a restricted game situation, where Alice and Bob are restricted to play the classical strategies, i.e., $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$, however Charlie is allowed to play the quantum strategies as well. It is shown that the quantum player is always better off for all values of the decoherence parameter p for increasing values of the memory parameter μ . It is seen that for the maximum degree of correlations, the effect of decoherence does not vanish in comparison to the two-player prisoner's dilemma quantum game. The three-players game does not become noiseless game which is in contradiction to the two-player case. It is also seen that for the maximum degree of memory, i.e. $\mu = 1$, that the quantum player can outscore the classical players for an entire range of the decoherence parameter p . The payoffs reduction due to the decoherence is controlled by the memory parameter throughout the course of the game. Furthermore, it is shown that the memory has no effect on the Nash equilibrium of the game.

References

- [1] Zurek W H 1999 *Phys. Today* **44** 36
- [2] Macchiavello C and Palma G M 2002 *Phys. Rev. A* **65** 050301
- [3] Lidar D A and Whaley K B 2003 *Springer Lecturer Notes in Physics* vol 622, ed F Benatti and R Floreanini (Berlin: Springer) p 83
- [4] Preskill J 1998 *Proc. Roy. Soc. Lond. A* **454** 385
- [5] Flitney A P and Abbott D 2002 *Fluct. Noise Lett.* **2** R175
- [6] Lee C F and Johnson N 2002 *Phys. Rev. A* **67** 022311
- [7] Du *et al* 2003 *J. Phys. A: Math. Gen.* **36** 6551
- [8] Eisert J and Wilkens M 2000 *J. Mod. Opt.* **47** 2543

- [9] Du J *et al* 2001 *Phys. Lett. A* **289** 9
- [10] Iqbal A and Toor A H 2002 *Phys. Lett. A* **300** 537
- [11] Meyer D A 1999 *Phys. Rev. Lett.* **82** 1052
- [12] Eisert J and Wilkins Mand Lewenstein M 1999 *Phys. Rev. Lett.* **83** 3077
- [13] Marinatto L and Weber T 2000 *Phys. Lett. A* **272** 291
- [14] Flitney A P and Abbott D 2002 *Fluct. Noise Lett.* **2** R175
- [15] Piotrowski E W and Sladkowski J 2002 *Physica A* **312** 208
- [16] Piotrowski E W and Sladkowski J 2003 *Int. J. Theor. Phys.* **42** 1089
- [17] Zhou L and Kuang L M 2003 *Phys. Lett. A* **315** 426
- [18] Du J, Li H, Xu X, Shi M, Wu J, Zhou X and Han R 2002 *Phys. Rev. Lett.* **88** 137902
- [19] Prevedel R, Stefanov A, Walther P and Zeilinger A 2007 *New J. Phys.* **9** 205
- [20] Chen K L, Ang H, Kiang D, Kwek L C and Lo C F 2003 *Phys. Lett. A* **316** 317
- [21] Nawaz A and Toor A H 2006 *J. Phys. A: Math. Gen.* **39** 9321
- [22] Ramzan M, Nawaz A, Toor A H and Khan M K 2008 *J. Phys. A: Math. Theor.* **41** 055307
- [23] Gawron P and Sladkowski J 2008 *Int. J. Quantum Info.* **6** 667
- [24] Benjamin S C and Hayden P M 2001 *Phys. Rev. A* **64** 030301
- [25] Du J F *et al* 2002 *Phys. Lett. A* **302** 229
- [26] Du J F *et al* 2002 *Fluct. Noise Lett.* **2** R189
- [27] Flitney A P and Abbott D 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** S860
- [28] Cao S *et al* 2007 *Chin. Phys.* **16** 915
- [29] Kay R, Johnson N F and Benjamin S C 2001 *J. Phys. A: Math. Gen.* **34** L547
- [30] Yeo Ye and Andrew Skeen 2003 *Phys. Rev. A* **67** 064301
- [31] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press)
- [32] Flitney A P and Abbot D 2005 *J. Phys. A: Math. Gen.* **38** 449
- [33] Karimipour V and Memarzadeh L 2006 *Phys. Rev. A* **74** 062311